For Publication in the IEEE Transactions on Reliability and Quality Control.

N65-88868
~~X63 16040~~

13p.

Code 2A

OPTIMUM RELIABILITY OF FOUR-SWITCH CIRCUITS

by

William A. Campbell [1963] [illegible] ~~[illegible]~~ Submitted for Publication

NASA, Langley Research Center, Langley Station, Va.
~~Langley Field, Virginia~~

OPTIMUM RELIABILITY OF FOUR-SWITCH CIRCUITS

16040

SUMMARY

A reliability analysis has been made of the ten possible ways to wire four switches between two terminals, and the results have been reduced to a form such that the most reliable configuration can be selected readily if the probabilities of open and short circuit failure are known. Four element redundancy was investigated because this level is necessary in order to achieve the required reliability of missile ignition systems when consideration is given to the fact that real switch elements such as relays and pressure switches have two modes of failure: failure to operate on command and failure by operating prematurely. This work was undertaken in order to determine the optimum circuitry for the RAM ignition system, but it is presented for general applications.

AUTHOR

INTRODUCTION

The success of many systems, present missile ignition systems for example, is largely dependent upon the successful operation of many relays and switches which cause the completion or disruption of any number of vital circuits. In considering the use of switches in any such system it is desirable to use the most reliable arrangement possible.

Associated with every normally-open switch is a probability of failure due to a number of events that may cause it to fail to close on command. There is likewise a probability of failure due to a number of events that may cause it to close prematurely. These probabilities will be referred to as Q_o and Q_s respectively. This paper will refer only to normally-open switches,but the results also can be used for normally-closed switches by merely redefining open and short circuit failures.

In order to insure against failure of a system due to a switch failure, two switches may be used instead of one. Two identical switches in series provide redundancy for a switch failure due to premature closing but do not safeguard against system failure due to a switch failing to close on command. On the other hand, these same two switches in parallel provide redundancy for a switch failure to close on command, but the circuit still fails if one of the switches closes prematurely.

There are four ways in which three switches can be wired in series, parallel, or series-parallel arrangements as shown in Figure 1. It is readily seen that none of these circuits provide redundancy for both of the above mentioned modes of failure. In order to get such a combination it is necessary to resort to a combination of four switches. There are only ten different ways in which four switches can be wired between two terminals, as shown in Figure 2.

Of these ten combinations, only two circuits, numbered 5 and 10 in Figure 2, provide complete redundancy for both modes of failure.

In other words, no one switch failure in either mode, can cause circuits 5 and 10 to fail. Because of this fact it would appear that one of these two circuits would be the most reliable four-switch system for any situation. However, it is not readily apparent whether any one of the other eight circuits is more reliable for any given probabilities of failure, Q_o and Q_s. Therefore, the object of this investigation was to compare all ten configurations for all values of Q_o and Q_s ranging from 10^{-12} to 1.0. This range is believed to cover all values encountered in any currently used switches.

THEORY

Each of the four-switch circuits may fail in either of two ways: one or more of the individual switches may close prematurely causing a short circuit across the system terminals, and one or more switches may fail to close on command causing all available paths across the terminals to remain open. The number and mode of individual switch failures necessary to fail the circuit depends upon the circuit configuration.

Assuming that an open circuit failure and a short circuit failure are mutually exclusive events, the total probability that the circuit will fail by either open or short circuit is

$Q_n = Q_{ot} + Q_{st}$ where

Q_{ot} is the probability that all available paths across the system terminals remain open after the close command, and Q_{st} is the probability of having a short circuit develop across the terminals prior to the close command.

The total probability of failure for each of the ten switch arrangements then is found by analyzing each circuit for series and parallel paths and determining Q_{ot} and Q_{st}. These probabilities are found by applying the basic probability laws governing series and parallel events. (In calculating these probabilities it was assumed that all switches are identical, i.e., all have the same Q_o and Q_s). For example, circuit 6 has all four switches in parallel. For this circuit to fail by open circuit, all four switches must fail to close on command. The probability of this is

$$Q_{ot} = (Q_o)\ (Q_o)\ (Q_o)\ (Q_o) = Q_o^4$$

For this circuit to fail by short circuit, only one of the switches has to close prematurely. The probability of this event is

$$Q_{st} = (Q_s + Q_s - Q_s^2) + (Q_s + Q_s - Q_s^2) - (Q_s + Q_s - Q_s^2)^2$$
$$= (4Q_s - 6Q_s^2 + 4Q_s^3 - Q_s^4)$$

Thus the total probability of failure of circuit 6 is

$$Q_6 = Q_{ot} + Q_{st} = (Q_o^4 + 4Q_s - 6Q_s^2 + 4Q_s^3 - Q_s^4)$$

The equations for each of the ten configurations, derived in the above manner, are presented in Figure 2.

PROCEDURE

In order to compare the probability of failure of all ten circuits, Q_o was assigned values from 10^{-12} to 1.0 in factors of ten.

For each value of Q_o, Q_s was varied from 10^{-12} to 1.0 in factors of ten for each circuit. The values of Q_n for all of these points were determined with the aid of an IBM 1620 computer.

Analysis of the computer data showed that one circuit proved most reliable for a certain range of values of Q_o and Q_s up to a point, and then another circuit would appear to be the most reliable for a range of values.

Figure 3 shows that there are boundaries between circuits 1 and 3, 3 and 5, 5 and 10, 10 and 8, and 8 and 6. In between these boundaries lie regions in which one circuit is more reliable than any other circuit. Figure 4 is a similar diagram showing the most reliable circuit for given values of Q_o and the ratio Q_o/Q_s.

RESULTS

The results show that there are ranges of values of Q_o and Q_s where any one of six of the circuits is the most reliable, although circuits 5 and 10 are best for the largest majority of cases. This was an expected result since these circuits are the only ones in which no one switch failure can cause circuit failure. It is interesting to note that when Q_o equals Q_s for the switches, both circuits 5 and 10 are equally reliable and either is better than any of the other configurations. In this case, circuit 5 would probably be chosen because it involves less wiring than circuit 10.

Since most all switching devices will not have ratios of Q_o/Q_s or Q_s/Q_o over 100, the area of prime interest in Figure 4 is the hatched-in area. It is seen that this area is made up predominantly of cases where circuits 5 and 10 are the most reliable.

However, it includes some cases where circuits 1, 3, 6 and 8 are best, and these circuits should be used in these cases. For example, suppose $Q_s = 0.01$ and $Q_o = 0.158$. These figures would correspond to failure rates of $Z_o = 170$ failures per million hours and $Z_s = 10$ failures per million hours for a mission time of 1000 hours, using the exponential failure law. Figures 3 and 4 show that circuit 8 is the most reliable circuit in this case.

It was found that for every combination of Q_o and Q_s circuits 2, 4, 7 and 9 were always less reliable than some other circuit.

CONCLUSIONS

In summary, the results of this report show that there are ten possible ways in which to arrange four switches in series, parallel, or series-parallel combinations. No one combination is best for all possible cases, but there are cases where, for given values of Q_o and Q_s, any one of six of the combinations could be the best system. For very small Q_o and Q_s, circuits 5 and 10 are generally, but not exclusively, better, circuit 5 being best when $Q_s > Q_o$, and circuit 10 being best when $Q_s < Q_o$. When $Q_o \gg Q_s$, circuit 6 is generally best, and when $Q_s \gg Q_o$, circuit 1 is generally best. However, there are regions of values of Q_o and Q_s where circuits 3 and 8 are best.

So, in order to determine the most reliable arrangement of four switches, it is necessary only to determine Q_o and Q_s, and enter Figure 3 with these values. The intersection of these two points will lie in a region defining the most reliable of the ten possible switch configurations.

WAC.sft

(Typed 9/3/63)

LIST OF SYMBOLS

Q_o - Probability of failure of a switch due to its failure to close on command (open circuit failure).

Q_s - Probability of failure of a switch due to its closing prematurely (short circuit failure).

Q_{ot} - Probability that a four-switch circuit will fail due to its failure to close on command.

Q_{st} - Probability that a four-switch circuit will fail due to a short circuit across the combination.

Q_n - Probability that the four-switch circuit number n will fail by either open or short circuit.

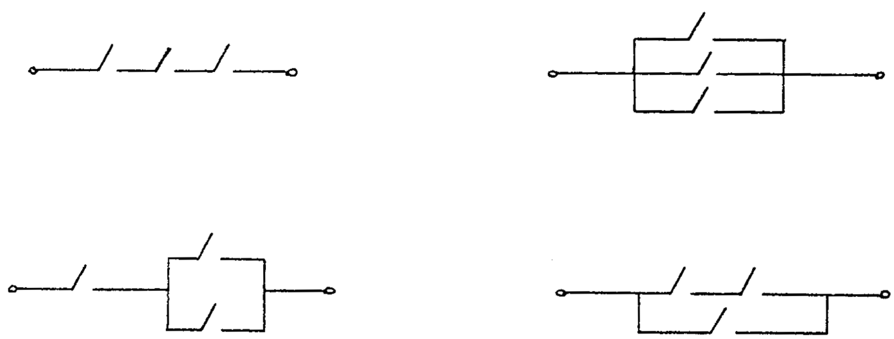
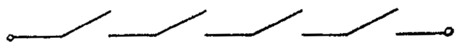


Figure 1. All possible arrangements of three switches.

FIGURE 2

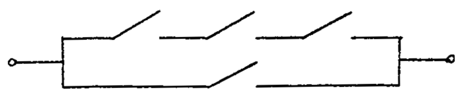
TEN WAYS TO ARRANGE FOUR SWITCHES

1)



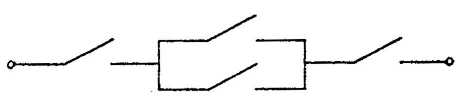
$Q_1=4Q_o-6Q_o^2+4Q_o^3-Q_o^4+Q_s^4$

2)



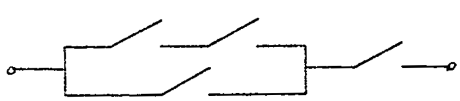
$Q_2=Q_s+3Q_o^2-3Q_o^3+Q_s^3+Q_o^4-Q_s^4$

3)



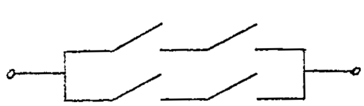
$Q_3=2Q_o-2Q_o^3+2Q_s^3-Q_s^4+Q_o^4$

4)



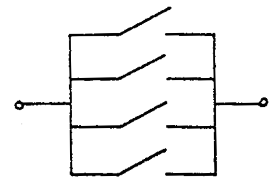
$Q_4=Q_o+2Q_o^2+Q_s^2+Q_s^3-3Q_o^3+Q_o^4-Q_s^4$

5)



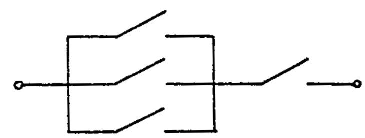
$Q_5=2Q_s^2+4Q_o^2-4Q_o^3+Q_o^4-Q_s^4$

6)



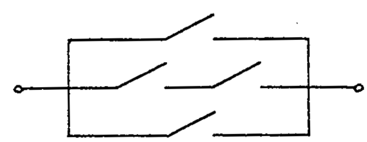
$Q_6=4Q_s-6Q_s^2+4Q_s^3-Q_s^4+Q_o^4$

7)



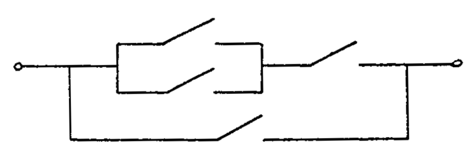
$Q_7= Q_o+3Q_s^2-3Q_s^3+Q_o^3+Q_s^4-Q_o^4$

8)



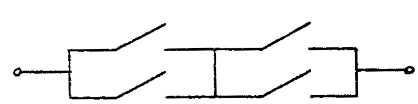
$Q_8=2Q_s-2Q_s^3+2Q_o^3-Q_o^4+Q_s^4$

9)



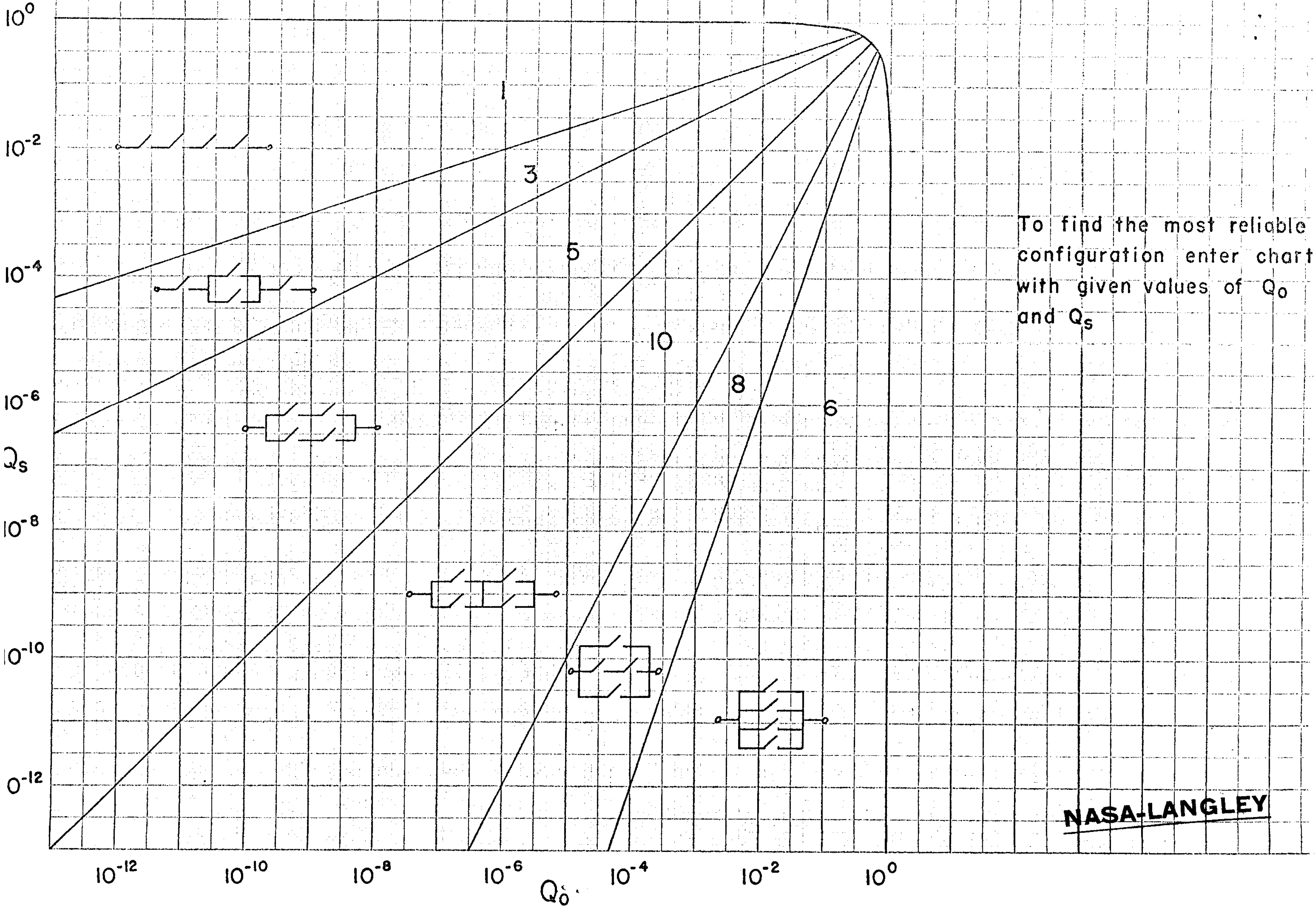
$Q_9=Q_s+2Q_s^2+Q_o^2+Q_o^3-3Q_s^3-Q_o^4+Q_s^4$

10)



$Q_{10}=2Q_o^2+4Q_s^2-4Q_s^3+Q_s^4-Q_o^4$

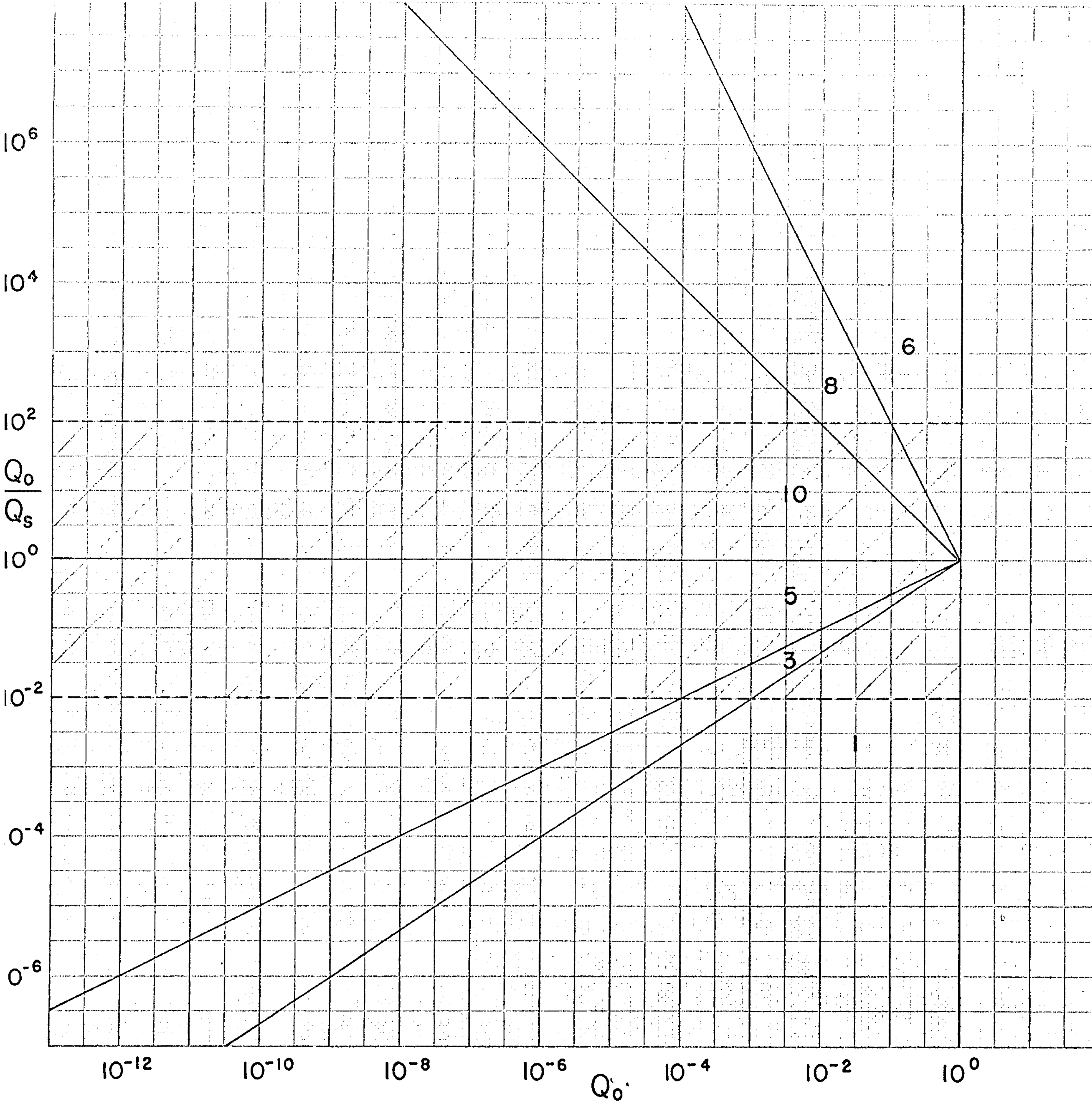
FIGURE 3



To find the most reliable configuration enter chart with given values of Q_o and Q_s

NASA-LANGLEY

FIGURE 4



To find the most reliable configuration enter chart with given values of Q_o and Q_o/Q_s.

NASA-LANGLEY

KEUFFEL & ESSER CO. N